C4 Differentiation

1. June 2010 qu. 2

Given that $y = \dfrac{\cos x}{1 - \sin x}$, find $\dfrac{dy}{dx}$, simplifying your answer. [4]

2. June 2010 qu. 5

Find the coordinates of the two stationary points on the curve with equation

$$x^2 + 4xy + 2y^2 + 18 = 0.$$ [7]

3. June 2010 qu. 7

The parametric equations of a curve are $x = \dfrac{t+2}{t+1}$, $y = \dfrac{2}{t+3}$.

(i) Show that $\dfrac{dy}{dx} > 0$. [6]

(ii) Find the cartesian equation of the curve, giving your answer in a form not involving fractions. [5]

4. Jan 2010 qu. 6

A curve has parametric equations $x = 9t - \ln(9t)$, $y = t^3 - \ln(t^3)$.

Show that there is only one value of t for which $\dfrac{dy}{dx} = 3$ and state that value. [6]

5. Jan 2010 qu. 7

Find the equation of the normal to the curve $x^3 + 2x^2y = y^3 + 15$ at the point (2, 1), giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. [8]

6. Jan 2010 qu. 8

(i) State the derivative of $e^{\cos x}$. [1]

(ii) Hence use integration by parts to find the exact value of $\displaystyle\int_0^{\frac{1}{2}\pi} \cos x \sin x \, e^{\cos x} \, dx$. [6]

7. June 2009 qu. 4

(i) Differentiate $e^x(\sin 2x - 2\cos 2x)$, simplifying your answer. [4]

(ii) Hence find the exact value of $\displaystyle\int_0^{\frac{1}{4}\pi} e^x \sin 2x \, dx$.

[3]

8. June 2009 qu. 5

A curve has parametric equations $x = 2t + t^2$, $y = 2t^2 + t^3$.

(i) Express $\dfrac{dy}{dx}$ in terms of t and find the gradient of the curve at the point (3, –9). [5]

(ii) By considering $\dfrac{y}{x}$, find a cartesian equation of the curve, giving your answer in a form not involving fractions. [4]

9. June 2009 qu. 8

(i) Given that $14x^2 - 7xy + y^2 = 2$, show that $\dfrac{dy}{dx} = \dfrac{28x - 7y}{7x - 2y}$. [4]

(ii) The points L and M on the curve $14x^2 - 7xy + y^2 = 2$ each have x-coordinate 1. The tangents to the curve at L and M meet at N. Find the coordinates of N. [6]

10. Jan 2009 qu. 6

A curve has parametric equations

$$x = t^2 - 6t + 4, \qquad y = t - 3.$$

Find

(i) the coordinates of the point where the curve meets the x-axis, [2]

(ii) the equation of the curve in cartesian form, giving your answer in a simple form without brackets, [2]

(iii) the equation of the tangent to the curve at the point where $t = 2$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. [5]

11. Jan 2009 qu. 8

The equation of a curve is $x^3 + y^3 = 6xy$.

(i) Find $\frac{dy}{dx}$ in terms of x and y. [4]

(ii) Show that the point $(2^{\frac{4}{3}}, 2^{\frac{5}{3}})$ lies on the curve and that $\frac{dy}{dx} = 0$ at this point. [4]

(iii) The point (a, a), where $a > 0$, lies on the curve. Find the value of a and the gradient of the curve at this point. [4]

12. June 2008 qu. 3

The equation of a curve is $x^2y - xy^2 = 2$.

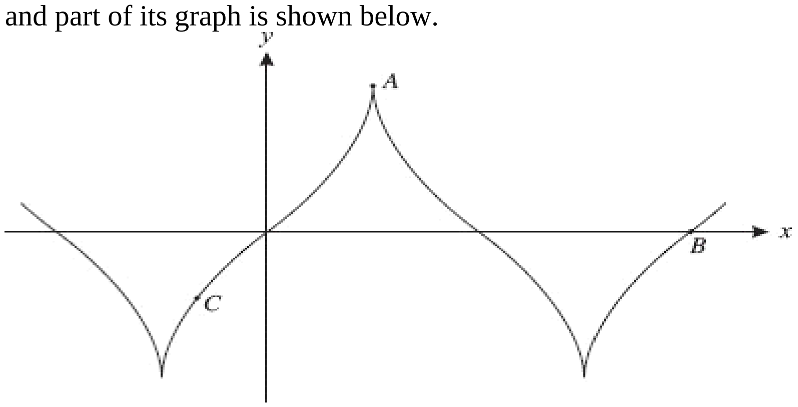
(i) Show that $\frac{dy}{dx} = \frac{y^2 - 2xy}{x^2 - 2xy}$. [3]

(ii) (a) Show that, if $\frac{dy}{dx} = 0$, then $y = 2x$. [2]

(b) Hence find the coordinates of the point on the curve where the tangent is parallel to the x-axis. [3]

13. June 2008 qu. 9

The parametric equations of a curve are $x = 2\theta + \sin 2\theta$, $y = 4\sin\theta$,
and part of its graph is shown below.



(i) Find the value of θ at A and the value of θ at B. [3]

(ii) Show that $\frac{dy}{dx} = \sec\theta$. [5]

(iii) At the point C on the curve, the gradient is 2. Find the coordinates of C, giving your answer in an exact form. [3]

14. Jan 2008 qu. 4

Find the equation of the normal to the curve $x^3 + 4x^2y + y^3 = 6$
at the point (1, 1), giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. [6]

15. Jan 2008 qu. 9

The parametric equations of a curve are $x = t^3$, $y = t^2$.

(i) Show that the equation of the tangent at the point P where $t = p$ is $3py - 2x = p^3$. [4]

(ii) Given that this tangent passes through the point $(-10, 7)$, find the coordinates of each of the three possible positions of P. [5]

16. June 2007 qu. 6

The equation of a curve is $x^2 + 3xy + 4y^2 = 58$. Find the equation of the normal at the point (2, 3) on the curve, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. [8]

17. Jan 2007 qu. 6

The equation of a curve is $2x^2 + xy + y^2 = 14$. Show that there are two stationary points on the curve and find their coordinates. [8]

18. June 2006 qu. 1

Find the gradient of the curve $4x^2 + 2xy + y^2 = 12$ at the point (1, 2). [4]

19. June 2006 qu. 9

A curve is given parametrically by the equations $x = 4\cos t$, $y = 3\sin t$, where $0 \le t \le \frac{1}{2}\pi$.

(i) Find $\frac{dy}{dx}$ in terms of t. [3]

(ii) Show that the equation of the tangent at the point P, where $t = p$, is $3x\cos p + 4y\sin p = 12$. [3]

(iii) The tangent at P meets the x-axis at R and the y-axis at S. O is the origin.

Show that the area of triangle ORS is $\frac{12}{\sin 2p}$. [3]

(iv) Write down the least possible value of the area of triangle ORS, and give the corresponding value of p. [3]

20. Jan 2006 qu. 2

Given that $\sin y = xy + x^2$, find $\frac{dy}{dx}$ in terms of x and y. [5]

21. Jan 2006 qu. 5

A curve is given parametrically by the equations $x = t^2$, $y = 2t$.

(i) Find $\frac{dy}{dx}$ in terms of t, giving your answer in its simplest form. [2]

(ii) Show that the equation of the tangent to the curve at $(p^2, 2p)$ is $py = x + p^2$. [2]

(iii) Find the coordinates of the point where the tangent at (9, 6) meets the tangent at (25, –10). [4]

22. June 2005 qu. 6

The equation of a curve is $xy^2 = 2x + 3y$.

(i) Show that $\frac{dy}{dx} = \frac{2 - y^2}{2xy - 3}$. [5]

(ii) Show that the curve has no tangents which are parallel to the y-axis. [3]

23. June 2005 qu. 7

A curve is given parametrically by the equations $x = t^2$, $y = \frac{1}{t}$.

(i) Find $\frac{dy}{dx}$ in terms of t, giving your answer in its simplest form. [3]

(ii) Show that the equation of the tangent at the point $P\,(4, -\frac{1}{2})$ is $x - 16y = 12$. [3]

(iii) Find the value of the parameter at the point where the tangent at P meets the curve again. [4]